MULTIPLICATION OF COMPLEX NUMBERS IN POLAR FORM

Rule: $r_1\ cis\ \theta_1 \ \times\ r_2\ cis\ \theta_2 = r_1 r_2\ cis\ (\theta_1 + \theta_2)$ multiply the moduli, and add the arguments

Proof: $r_1\ cis\ \theta_1 \ \times\ r_2\ cis\ \theta_2$

$$
\begin{aligned}
&= r_1(\cos\theta_1 + i\sin\theta_1) \times r_2(\cos\theta_2 + i\sin\theta_2)\\
&= r_1 r_2(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)\\
&= r_1 r_2(\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2)\\
&= r_1 r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2))\\
&= r_1 r_2(\quad \cos(\theta_1+\theta_2) \quad + \quad i(\sin(\theta_1+\theta_2)) \quad)\\
&= r_1 r_2\ cis\ (\theta_1 + \theta_2)
\end{aligned}
$$

Line 4 to Line 5 uses these Trig Identities to convert the trig expressions:
- cos (A+B) = cos A cos B – sin A sin B
- sin (A+B) = sin A cos B + cos A sin B

Example: Calculate $5\ cis\ 60° \times 2\ cis\ 150°$.
Leave your answer in polar form.

$$
\begin{aligned}
&5\ cis\ 60° \times 2\ cis\ 150°\\
&= 5 \times 2\ cis\ (60° + 150°)\\
&= 10\ cis\ 210°\\
&= 10\ cis(-150°)
\end{aligned}
$$

convert θ to its principal value(which is within $-180° \le \theta \le 180°$)

DIVISION OF COMPLEX NUMBERS IN POLAR FORM

Rule: $\frac{r_1 cis\ \theta_1}{r_2 cis\ \theta_2} = \frac{r_1}{r_2}\ cis\ (\theta_1 - \theta_2)$ divide the moduli, and subtract the arguments

Proof: similar to above, multiply the top and bottom of the quotient by the conjugate of the denominator. Simplify using the trig identities.

Example: Calculate $\frac{16\ cis\ 30°}{2\ cis\ 90°}$. Leave your answer in polar form.

$$\frac{16\ cis\ 30°}{2\ cis\ 90°} = \frac{16}{2}\ cis\ (30° - 90°) = 8\ cis\ (-60°)$$

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